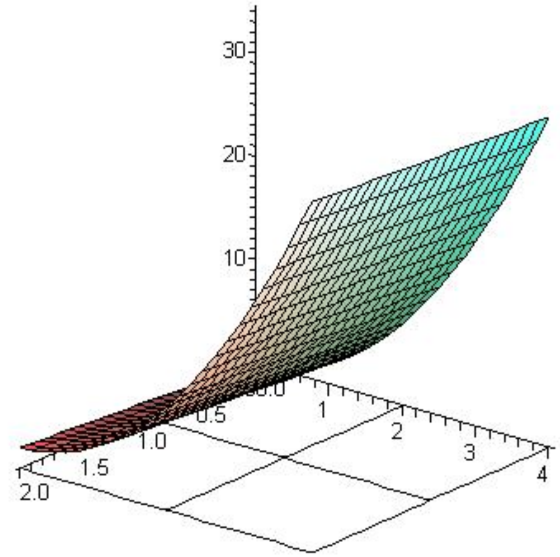
Closing Tue: 15.1, 15.2
Closing Thu: 15.3, 15.4
Midterm 2 is Tuesday, March 1
It covers 13.3/4, 14.1/3/4/7, 15.1-15.4



15.1 & 15.2 Double Integrals over Rectangles
Goal: Give a definition for volume “under” a surface and write this volume in terms of integrals.

Example:
Consider the volume under the surface
$z = f(x,y) = x + 2y^2$ and above the rectangle
$R = [0,2] \times [0,4] = \{(x,y) : 0 \leq x \leq 2, 0 \leq y \leq 4\}$

Let’s approximate this volume.
(a) Draw the region R in the xy plane and break it into 4 sub-regions;
m = 2 columns and n = 2 rows.
(b) Approximate using a rectangular box over each region.

In general, we define:

$$\iint_R f(x,y)dA = \lim_{m,n\to\infty} \sum_{i=1}^{m} \sum_{j=1}^{n} f(x_{ij}, y_{ij})\Delta A$$

= the `signed' volume between *f(x,y)* and the *xy*-plane over *R*.

If *f(x,y)* is above the xy-plane it is positive.

If *f(x,y)* is below the xy-plane it is negative.

General Notes and Observations:

z = $f(x,y)$ = height on surface

R = the region on the *xy*-plane

ΔA = area of base = $\Delta x \Delta y = \Delta y \Delta x$

$f(x_{ij}, y_{ij})\Delta A$ = (height)(area of base)

= volume of one approximating box

Units of $\iint_R f(x,y)dA$ are

(units of f(x,y))(units of x)(units of y)

Other quick applications:

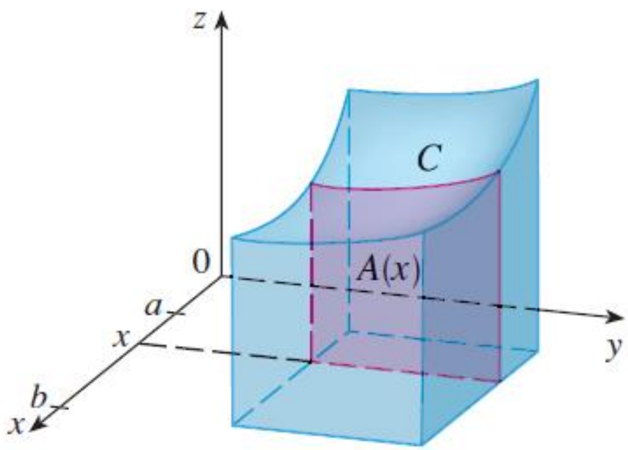
$$\iint_R 1dA = \text{Area of R} \quad \text{and}$$

$$\frac{1}{\text{Area of R}} \iint_R f(x,y)dA = \text{Average value of f(x, y) over R}$$

15.2 Using Iterated Integrals to Compute

If you fix x: The area under this curve is given by

$$\int_c^d f(x,y)dy = \text{cross} - \text{sectional area under surface at this fixed value of } x$$



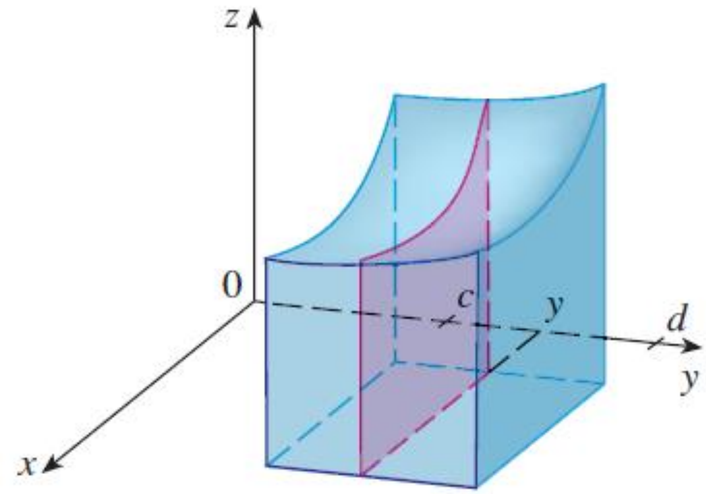
From Math 125,

$$\text{Volume} = \int_a^b \text{Area(x) } dx$$
$$= \int_a^b \left(\int_c^d f(x,y)dy \right) dx$$

We can also do cross-sections the other direction.

If you fix y: The area under this curve is given by

$$\int_a^b f(x,y)dx = \text{cross} - \text{sectional area under surface at this fixed value of } y$$



$$\text{Volume} = \int_c^d \text{Area}(y)dy$$
$$= \int_c^d \left(\int_a^b f(x,y)dx \right) dy$$

Three Problems Like 15.2 homework:

1. Find the volume under z = x+ $2y^2$ and above the rectangular region

$$0 \le x \le 2, \quad 0 \le y \le 4$$

2. $\int_0^3 \int_0^1 2xy\sqrt{x^2 + y^2}\,dxdy$

3. Find the double integral of *f(x,y) = y cos(x+y)* over the rectangular region

$$0 \le x \le \pi, \quad 0 \le y \le \pi/2$$